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Chaos in Discrete Production Systems?

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Abstract

In literature, several cases are reported of models of discrete non-stochastic production systems that show irregular, apparently chaotic behavior. In this paper a number of these cases are analyzed, and the irregular behavior is attributed to (1) chaotic behavior in hybrid models, (2) chaotic behavior in discrete-event models that use a chaotic map, or (3) periodic behavior with a period longer than the observation window. The irregular behavior of a discrete-event model of a two-machine production system is analyzed by means of nonlinear time series analysis and sensitivity analysis. This case reveals the possibilities and limitations of the application of chaos theory to discrete-event models of production systems. Also, a new method for determining the sensitivity of discrete-event models to truly small changes is introduced. Realistic, non-artificial discrete-event models of discrete production systems that show chaotic behavior were not found in this study.

Keywords: *Chaos, Production Systems, Discrete-Event Models, Simulation, Time Series Analysis, Sensitivity Analysis*

1 Introduction

In production systems, variability is a well-known factor with many negative effects. Variability can be divided into process variability, where processing times of workstations or operators are variable, and flow variability, where the inter-arrival times of products arriving at workstations are variable. Queueing time increases proportionally for both process variability and flow variability. Therefore, high variability leads to long cycle-times, high WIP-levels, wasted capacity and lost throughput [1]. The impact of variability is magnified if the utilization of a system is very high. According to [1], the most frequent causes of variability are:

- natural variability, which includes minor fluctuations in process time due to differences in operators, machines, and material;
- random outages;
- setups;
- operator availability;

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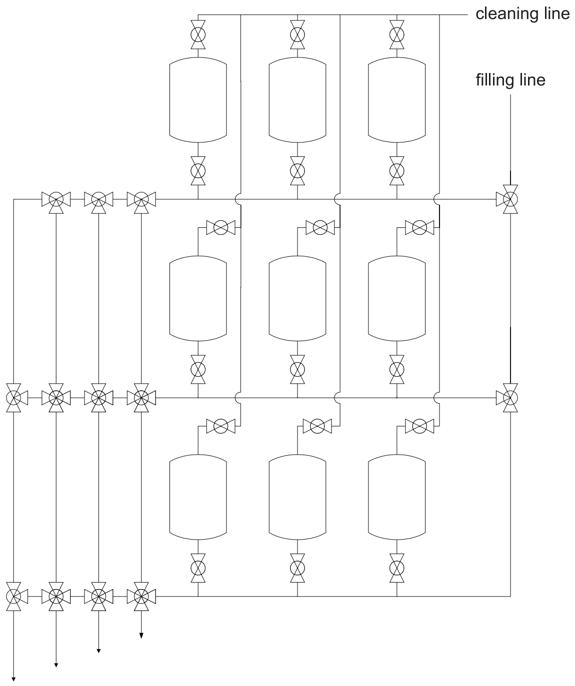


Figure 1: A cluster of nine tanks on three connection pipes and one manifold.

- recycle.

The time-dependent behavior of production systems is often analyzed by means of simulation models. In this way, flow variability of systems that are too complex for mathematical analysis can be determined. The Systems Engineering Group of the Eindhoven University of Technology makes extensive use of simulation models for the design of advanced industrial systems. For this purpose, the χ (Chi) formalism is used [2, 3, 4]. The application field is intended to be very wide: ranging from chemical or mechanical physical systems [3] to complete productions systems, such as a fruit-juice production plant [5] or an integrated circuit manufacturing plant [6]. Control systems that are modeled in χ range from control systems of individual machines [7], to control systems based on complex scheduling rules [8]. For simulation based testing, these control system models can be connected to models that specify the behavior of the controlled systems.

In χ discrete-event simulation models of a part of a brewery, high flow variability was observed even though the models did not contain any outages, nor setups, nor operators (fully automated system), nor recycles; and all processing times were constant. The part of the plant consists of some 70 high volume tanks, that are interconnected by a complex network of pipes, valves and pumps. It is operated at maximum level: whenever a batch is finished, and a tank is emptied, a new batch is allowed to enter the system. There are several shared resources in the system: many tanks share the same pipes, pumps, cleaning equipment, and processing equipment. The control system tries to achieve maximum throughput, and minimum waiting times for critical operations, by means of an advanced reactive scheduling strategy. The tanks are divided into several clusters, one of which is shown in Figure 1. Figure 2 shows the throughput results of an experiment with one of the simulation models for 5000 batches. The throughput (in batches per hour) is determined by means of a moving window of twenty-one batches, which is the average weekly production. Apart from the negative effects of variability on waiting times, cycle times, and throughput, the high variability in the throughput also makes it difficult to determine the actual throughput. Some

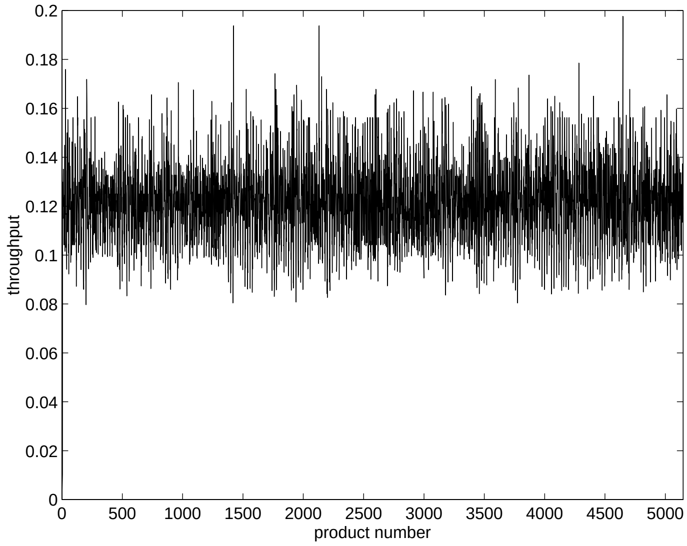


Figure 2: Throughput of a non-stochastic discrete-event model of a part of a brewery.

other configurations and scheduling strategies were simulated, but all models showed irregular, seemingly random throughput behavior.

Since a few decades, so called "chaotic systems" are known. These nonlinear deterministic systems show irregular, seemingly random behavior. The theory of chaotic systems is well established for systems of differential equations and for difference equations (e.g. [9, 10]), such as found in mechanical systems. Application of this theory to production systems is a relatively new area. The goal of the application of this theory is to characterize the irregular behavior of non-stochastic models of production systems and to understand the causes of this behavior. This may lead to knowledge about the system that can be used to reduce variability.

Production systems can be divided into two types: discrete production systems and production systems of the so-called process type. Discrete production systems can be characterized by the fact that discrete products need to be positioned. In production systems of the process type, on the other hand, there is no positioning of intermediate products. This is especially clear when materials are in gaseous, liquid or powder form. Models of production systems can be divided into continuous-time models, discrete-event models, and combined continuous-time / discrete-event models (also referred to as hybrid models) [3]. Discrete production systems are usually specified by means of discrete-event models. Process type production systems are usually specified by means of continuous-time or hybrid models. Some batch production systems of the process type, such as the brewery discussed above, may also be specified by means of discrete-event models.

In this paper, the theory of chaotic systems is applied to discrete production systems. In Section 2, literature on production system models that appear to exhibit chaotic behavior is discussed. These models are either hybrid models, models based on a chaotic map, or models that exhibit periodic behavior with a period longer than the observation window. The presence of chaos in a model of a production system, however, only implies chaos in a *real* discrete production system if the model is a *valid* model of the real discrete production system. In [11], validation is defined as the process of determining whether a simulation model is an accurate representation of the system, for the particular objectives of the study. In Section 3, the theory of chaos in dynamic systems is applied to small discrete-event models of production systems. It is shown that many nonlinear analysis methods cannot be used for the analysis of discrete-event models, or should be used with care—

especially in the case of sensitivity analysis. Also a new method is introduced for determining the sensitivity of discrete-event models to truly small changes. Finally, conclusions are presented in Section 4.

2 Related research

Chaos in models of *process type* production systems is treated in many papers. The models considered are usually hybrid models of a system of liquid tanks with a switched flow server. Chaos in models of *discrete* production systems that were found in literature can be divided into two categories: chaos in hybrid models of discrete production systems, and chaos in discrete-event production system models in which a chaotic map is used. The different kind of production system models that exhibit chaotic behavior are treated below in three subsections. In each subsection, some representative papers are treated. No attempt is made to be complete in this respect. Application of chaos control methods such as discussed in [12] is not considered. In the fourth subsection, papers are discussed in which irregular, apparently chaotic behavior of a discrete-event model is proved to be periodic instead of chaotic. In literature, discrete production systems are often classified as chaotic as a result of structural stability analysis. This is discussed in greater detail in Section 3, subsection 'Structural stability analysis'.

Chaos in hybrid models of process type production systems

The presence of chaos in hybrid models of production systems of the process type is well known. In [13, 14, 15, 16, 17], a hybrid model of a production system that consists of three tanks and a switched server (see Figure 3), is treated extensively. Fluid continuously flows out of each tank.

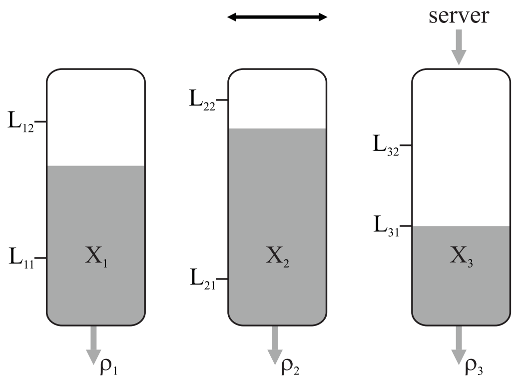


Figure 3: Hybrid system consisting of three tanks and a switched server.

To compensate for the fluid loss, the server can fill one tank at a time with a constant rate. Initially, the server is in position j. It remains in that position and fills tank j until the height of the fluid reaches the upper threshold L_{j2}, or another tank, say k, reaches its lower threshold L_{k1}. In the first case, the server stops filling tank j. In the second case, the server switches instantaneously to tank k, filling the tank until the next tank reaches its threshold, and so on. In [13], a model of differential equations is used to show by means of statistical analysis that the system exhibits chaotic behavior. In [16], a simulation experiment is performed. The data obtained is used to determine the entropy ($h = 0.644741$). In [17], a simulation model of the hybrid production system is built, using hybrid χ [3, 7]. Using the time series analysis methods from the TISEAN package [18], a positive Lyapunov exponent ($\lambda = 0.20$) and a fractal dimension ($d(\epsilon) = 3.2 \pm 0.2$) are found.

Chaos in hybrid models of discrete production systems

The difference between a discrete-event model and a hybrid model of a discrete production system can be quite subtle:

Consider a discrete-event model of a production system consisting of a single machine that produces discrete products. If the processing time of the machine is 1 time-unit, and the machine never needs to wait for the supplied material, then the number of products produced by the machine changes discontinuously at each discrete time-unit. If i is an integer value, and i^- is defined as the time just before time-point i, and i^+ is defined as the time just after time-point i, then the number of products produced is: 0 from time 0 to 1^-, 1 from time 1^+ to 2^-, etc. The throughput of the production system is the number of products produced divided by the required time. This means that the throughput at time 0 to 1^- is 0 products per time-unit; at time-point 1^+, the throughput equals 1; at time-point 2^-, the throughput equals 1/2, and at time-point 2^+, the throughput equals 1 again. At each discrete time-point, the throughput changes discontinuously. In between two adjacent discrete time-points, the throughput remains the same.

In a hybrid model of such a production system, the throughput of a machine with a processing time of d time-units can be approximated as a continuous flow of $1/d$ products per time-unit. This is in fact equivalent to a model of a liquid tank that has an outgoing flow of $1/d$. An example of such a model can be found in [19]. Another way to obtain a hybrid model of the production system described above is to add equations to the existing discrete-event model. Such an equation could, for example, define the throughput φ as a function of the time t: $\varphi(t) = n/t$, where n is the number of products produced so far. This approach is taken in [20], where the biggest Lyapunov exponent found equals 0.0137. This is quite close to zero, and could be the result of an insufficiently long time series. A different example of a hybrid model of a discrete production system is presented in [21]. In this paper, time-series obtained from a wafer fabrication plant are used to construct a best fitting 'black box' ARIMA model, which model is shown to exhibit chaotic behavior.

Chaos in production system models using a chaotic map

Chaotic behavior of a deterministic discrete-event model of a paint spraying installation is demonstrated in [22]. The paint is sprayed on the workpieces in several steps, such that the thickness of each coating depends on the previous coating, and on a parameter r. The workpieces undergo several cycles in the paint spraying installation. The thickness of each layer of coating of the workpiece is represented by $x_{n+1} = rx_n(1 - x_n)$, where x_i is the thickness of the added layer after i cycles through the installation. This equation is the well-known Logistic map, see e.g. [9, 10]. For the right setting of parameter r, the behavior of this map is chaotic.

In [23], a chaotic map is considered to be a model of a production cell. The system is modeled as a closed-loop system, see Figure 4. Here X_n is defined as "the quantity of parts present in

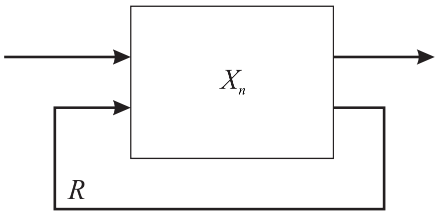


Figure 4: Model of a production system [23].

the cell", which is also referred to as WIP (Work In Process), and R is a constant. After some intermediate formulas, the behavior of this system is derived to be $X_{n+1} = X_n + X_n(R - X_n/X)$, where X (without index) denotes the desired WIP-level. This equation is again a version of the Logistic map, which exhibits chaotic behavior for the right settings of parameters. The value of X_{n+1} and X_n in the map are real values. However, since X_n is defined as the quantity of parts present in the cell, X_n must be an integer value. Experiments performed in [17] show that the necessary round off of the numbers leads to behavior that is not chaotic, but to behavior where X_n becomes 0, approaches $-\infty$, or becomes much bigger than the desired WIP-level—depending on the value of X_0, the value of R and the desired WIP-level.

Irregular discrete-event behavior proved to be periodic

In [24, 25], the irregular behavior of some small models of discrete production systems is proved to be periodic, not chaotic. The modeled production systems consist of two machines and three (or four) buffers. Products entering the system undergo three (or four) operations. After the first two operations, the products are recycled for the remaining operations. The number of products in the buffers is modeled by means of maps. The maps are shown to be periodic for certain combinations of the processing times of the machines in the system. The period, however, can be given any length by adjusting the processing times of the machines. When the length of the period goes to infinity, it becomes impossible to observe the period of the behavior.

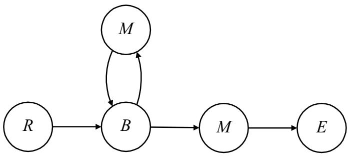
3 Application of chaos theory to discrete production systems

Models of production systems

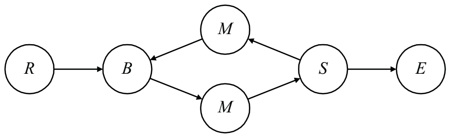
In this section, chaos theory and tools are applied to discrete-event models of discrete production systems. Time series analysis requires data that is *stationary*, and covers a stretch of time that is much longer than the longest characteristic time scale that is relevant for the evolution of the system [26]. Since it is extremely difficult to satisfy these requirements for real production systems, models of production systems are used. Time series analysis methods are very susceptible to noise. Therefore, non-stochastic models of production systems are used.

In the sequel, the term irregular behavior is used to indicate behavior that is neither stable nor periodic in the observed time interval. Clearly, in this definition, the classification of irregular behavior depends on the length of the observed time interval. In the experiments below, typically a time-interval equivalent to approximately 10^5 products is used.

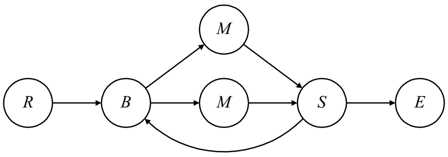
To investigate the presence and origin of chaotic behavior of discrete production systems, a number of models of small production systems have been developed. In particular, production systems with product recycles are expected to show irregular, and possibly chaotic behavior. In Figure 5, three basic models with product recycles are represented informally. The models consist of order release unit *R*, a buffer *B*, machines *M*, possibly a switch *S* and exit *E*. In [17], also several other small models are investigated for irregular behavior. This is done by changing the values of the following parameters: the processing times of the products in each machine and the Work In Process (WIP). The way that the values of these parameters are changed is inspired by "period doubling bifurcations", which may lead to chaos [9, 10]. The processing times and the WIP-level are changed one by one. After each change, a new simulation experiment is executed, and a time series of the interdeparture times of the first $2 \cdot 10^5$ products is generated. If this time series becomes periodic or stable (period of zero), the period is determined, otherwise the search is terminated. If the change leads to a longer period, the next change is made in the same direction. If the change



(a) System BMM



(b) System BMSM



(c) System BMMS

Figure 5: A collection of small discrete-event production system models.

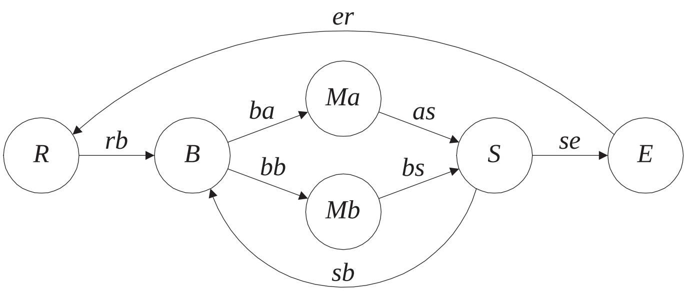


Figure 6: Simulation model of production system BMMS.

does not lead to a longer period, the value of another parameter is changed. If necessary, the step size of the change is adjusted.

For the models represented in Figure 5(a) and Figure 5(b), no irregular behavior could be found. Both models showed either stable or periodic behavior, with a period that is smaller than approximately $2 \cdot 10^4$ products. System BMMS, represented in Figure 5(c), does show irregular behavior.

The discrete-event simulation model of production system BMMS was developed in χ (Chi), a specification language for which simulation models can be obtained [2, 3, 4]. The model consists of order release unit *R*, buffer *B*, two parallel machines *Ma* and *Mb*, switch *S* and exit *E*, see Figure 6. Every product that enters the system undergoes two operations. Both operations can be performed by both machines; *Ma(1)* and *Ma(2)* are the respective processing times of the first and second operation on a product in machine *Ma*, whereas *Mb(1)* and *Mb(2)* are the respective processing times of machine *Mb*. Process *R* reads a list of products that are to be processed. If the Work In Process (WIP) level is below the maximal WIP, *R* sends a product to buffer *B* via channel *rb*. If there is a product in the buffer and a machine is empty, the buffer immediately sends a product to this machine. The product is selected from the buffer according to the FIFO (First In, First Out) scheduling rule. If both machines are available, the first selected product is sent to machine *Ma*, the second product is sent to *Mb*. When the processing time has passed, the product is sent to

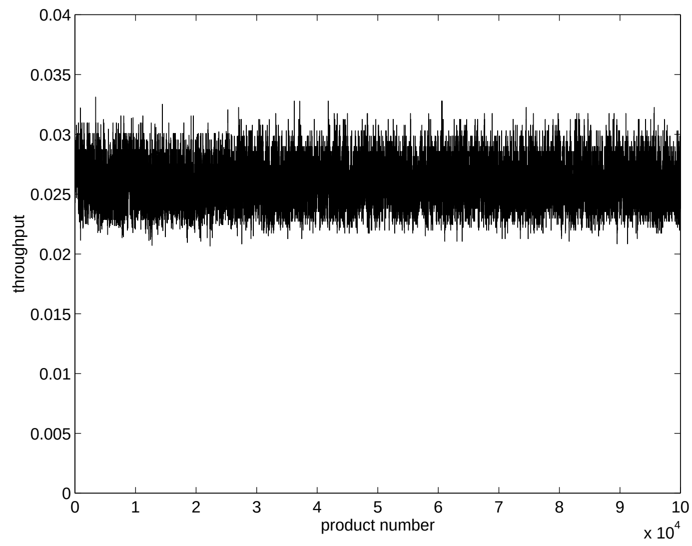


Figure 7: Throughput versus product number of system BMMS.

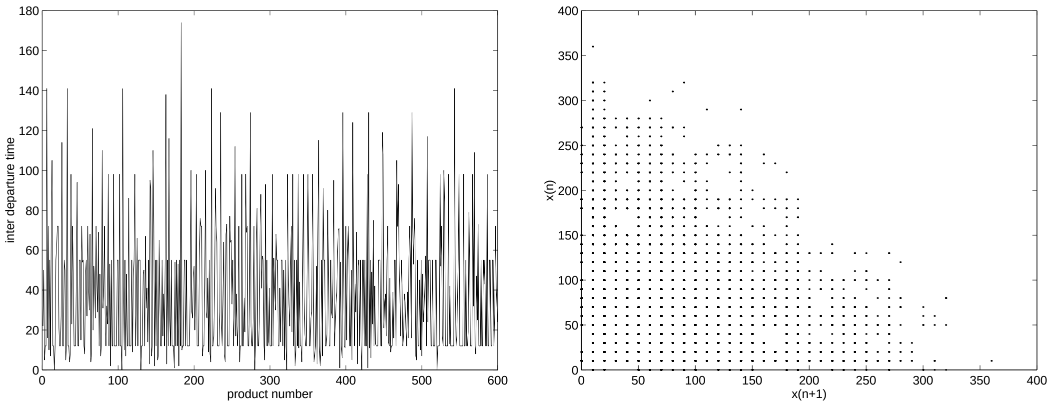
process *S*. If both machines are ready at the same time, machine *Ma* sends its product first. If the first operation has been performed, the product is sent back to buffer *B* via channel *sb*. If the second (and last) operation has been performed, the product is sent to process *E* via channel *se*. This process informs *R* that the WIP-level has decreased by one. A time stamp is added to a product when it is released by *R*, and when it arrives at *E*. These time stamps make it possible to determine the flow times of the products, the system interdeparture times and the throughput. The throughput (in number of products per unit of time) is determined using a moving window of 20 products, which is the WIP-level of the model. Figure 7 shows simulation output of this model for $Ma(1) = 43$, $Ma(2) = 12$, $Mb(1) = 57$, $Mb(2) = 72$, and $\text{WIP} = 20$. No periodic behavior was found for the first $2 \cdot 10^5$ products. Therefore, system BMMS is used to investigate if chaotic behavior can be determined, using nonlinear analysis methods.

Nonlinear analysis methods

When analysis methods using systems of equations are to be applied, valid models of production systems based on differential or difference equations are required. However, almost no valid models consisting of differential or difference equations are available. Only for very small, specific production systems, suitable and valid systems of equations have been developed [24]. For production system BMMS, no valid model consisting of differential or difference equations is available. Therefore, time series analysis is used instead.

The time series are generated by means of simulation model BMMS. For each experiment, a time series of 200 000 interdeparture times is generated. The interdeparture time (time between two finished products) is chosen, because of the fact that the interdeparture times of a production system are often the interarrival times for the next system. Interarrival patterns are important for the planning and control of a production system. Next to that, the interdeparture times are less correlated than the successive product flow times. This makes it easier to detect trends and correlations caused by the systems dynamics. To get rid of start-up effects, the first 10 000 points are not used.

To find a good embedding of the attractor of this system in phase space, the time lag ν and the



(a) Interdeparture times of 600 products, showing that a small number of values occur.

(b) Phase space projection of the interdeparture times of 200000 products, showing that a small number of states occur.

Figure 8: Interdeparture times and phase space projection of data from system BMMS.

embedding dimension m need to be determined [26]. The time series from a discrete-event system can be seen as map-like data. For this kind of data, the time lag is fixed to $\nu = 1$.

In order to determine the embedding dimension, an algorithm based on the false nearest neighbor method is used. This has been done by means of the TISEAN package [18], which implements the concepts described in [26]. The data in this time series, however, is not suitable for this method. The interdeparture times show an irregular pattern, but the number of different values they take is small. The cause of this small number of different values is that the processing times are constant, so that the flow times consist of sums and differences of these values. Therefore, the number of different interdeparture times is limited. The result of this limited number of values is that in the reconstructed phase space, neighboring points lie either far away, or exactly on the same spot.

The problem of the limited number of variables values is illustrated in Figure 8. In Figure 8(a), the time series for products 5000 to 5600 is given. It shows that the interdeparture times only take a small number of different values. Figure 8(b) shows a projection of the time series for 200 000 products in phase space on a plane. It is clear that only a small number of states occur. In reality, the amount of states may be more, because of the fact that some states may be projected onto one another, as a result of the embedding in a 2-dimensional phase space. The unknown optimal embedding dimension may be larger.

The result of this limited number of states in phase space is that the methods based on nearest neighbors or close returns are not effective anymore. This is caused by the fact that neighbors lie either far away, or exactly on the same spot. Therefore, neither the embedding dimension, nor the maximal Lyapunov exponent, nor the correlation dimension can be determined. As a result, characterization of the behavior of system BMMS, using these nonlinear time series analysis methods, is not possible.

The fact that the elements of the time series take a limited number of recurring values, appears to be quite general for data obtained from discrete-event models of production systems. This limited number of recurring values also arises in simulation experiments with other models, such as those displayed in Figure 5, and is also reported in [27]. In case of the hybrid model of a production system consisting of three tanks (see Section 2), the nonlinear analysis methods function correctly.

Sensitivity analysis

Since the discussed nonlinear time series analysis methods cannot be used to characterize the behavior of model BMMS, sensitivity analysis is used instead. One of the characteristics of chaos is sensitivity to small differences in the initial conditions. Sensitivity can be determined by perturbing a continuous variable with a small amount. If this leads to big differences in the two time series, the system is sensitive to this small perturbation. If there is no sensitivity, one of the defining characteristics of chaos is not present, so that the behavior of the model is not chaotic.

For continuous or hybrid production systems, sensitivity analysis is straightforward. One of the continuous variables of the system can be perturbed to realize a small difference in the initial conditions. In case of a discrete-event model of a production system, however, no continuous variables are present, apart from the time. The number of elements in a buffer, for example, can only be perturbed by one or more full units, such as done in [28]. Therefore, a new method is required for determining the sensitivity of discrete-event models to truly small changes.

In order to develop such a method, a parallel is drawn to hybrid production systems. A hybrid production system may consist of a number of tanks, that are connected with pipes. The continuous variable modeling the volume of the fluid in a tank can be perturbed for sensitivity analysis. This volume can be regarded as the "work content" at a certain time in this tank. Knowing the outflow, the fluid volume can be translated into an amount of time necessary to process this work content. The work content of a machine in a discrete-event system can be regarded as the "part" of the product that is being processed but not finished yet. In this way, it is possible to translate the work content into an amount of time: i.e. into the time necessary to finish the product that is processed in the machine. By adding (or subtracting) slightly different amounts of time to (or from) the time necessary to finish the product that is processed, experiments with a small difference in the work content can be compared.

Results of a sensitivity analysis of a discrete-event model may give a false indication of sensitivity. This is due to the fact that a sensitive reaction to a specific small change at a certain state of the model, need not be typical for the model. This is illustrated below by applying a sensitivity analysis to the discrete-event model of production system BMMS.

For the sensitivity analysis, three parameters are introduced in the simulation model of system BMMS. The first parameter is the machine for which the processing time is perturbed, the second parameter *np*, is the operation number of the selected machine for which the perturbation is applied; the operation number of each machine starts at one and is increased by one after each product that is processed by the machine. The third parameter, *tp*, is the small amount of time that is added to the processing time of the selected product (operation number).

The results of the simulation experiments on system BMMS are as follows. Figure 9 shows the flow times of the products for two experiments: for the dashed line with circles no perturbation is applied, the dotted line with plus marks is the result of an initial perturbation in the processing time of machine *Ma* for the first operation. The difference in both paths after approximately 50 products can easily be distinguished, which indicates sensitivity to this small change. To investigate if this behavior is general for this system, more experiments are performed. Table 1 shows the results of these experiments for 10000 finished products, $Ma(1) = 43$, $Ma(2) = 12$, $Mb(1) = 57$, $Mb(2) = 72$, and $\text{WIP} = 20$. In the table, for practical reasons, only the total time needed to process all products is displayed. A difference in this value shows that a difference occurred in the paths of the simulation; if there is no difference in the total times, the paths were (almost) the same. This was verified by plotting all experimental measurements. It is clear from Table 1 that "sensitivity" occurs only in specific states and for specific perturbations. Therefore, the discrete-event model of production system BMMS is not chaotic in the strict sense as defined in the well established theory

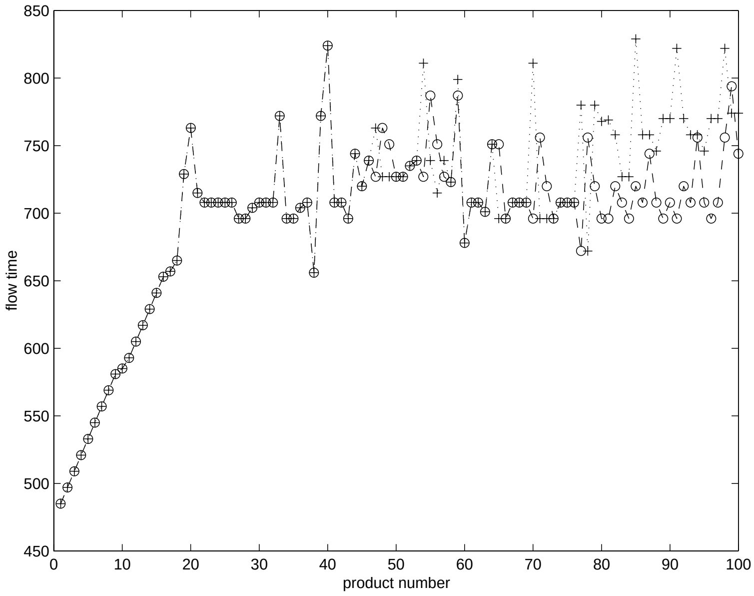


Figure 9: Sensitivity analysis showing the results of a change of 0.01 in the initial processing time of machine *Ma* (o = no change, + = change of 0.01).

Machine	*np*	*tp*	total processing time
Ma	0	0	385913
Ma	1	-0.01	385913
Ma	1	0.01	386043
Ma	1	1	386043
Ma	1	1.01	385535
Ma	1	1.5	385536
Mb	1	0.01	385913
Mb	1	-0.01	386043
Mb	1	-0.5	386042
Ma	3000	-0.01	385913
Ma	3000	0.01	383692
Ma	3000	0.5	383692

Table 1: Experiments to investigate sensitivity to small 'initial' differences, showing that system BMMS is sensitive only to specific small changes *tp* in the processing time after *np* products.

of chaos in dynamic systems [9, 10].

Structural stability analysis

When trying to find chaotic behavior, care should be taken not to confuse sensitivity analysis with structural stability analysis. Sensitivity analysis is often used to indicate chaotic behavior of production systems, since sensitive dependence on differences in the initial state is one of the defining characteristics of chaos. Small differences in the values of *variables* of a chaotic system diverge exponentially. A different method is the analysis of the structural stability of a dynamical system. In this method, *parameters* are perturbed to investigate qualitative changes in the behavior of the system. It is well known that changes in parameters such as scheduling rules and WIP-levels of a system can lead to qualitative changes in the behavior of a system.

To perform a sensitivity analysis for a production system, the initial value of one of the variables of the system should be changed with a small amount. Scheduling rules are parameters, not variables. Therefore, by changing these parameters, such as done in [28], a structural analysis is performed instead of a sensitivity analysis.

4 Conclusions

Discrete production systems may show irregular, seemingly random behavior. Even in models of production systems in which no stochastic phenomena are modeled, this behavior can be observed. The variability of the product flow has many negative effects, such as long cycle-times, high WIP-levels, and lost throughput. In order to get more insight in the nature of this irregular behavior, and in what can be done to prevent it from occurring, the behavior has been analyzed using the analysis methods and tools that were developed for the analysis of chaotic nonlinear dynamic systems.

Because of the difficulties to apply the nonlinear analysis methods directly to real industrial production systems, non-stochastic models are used. The smallest discrete-event model that was found of a production system exhibiting irregular behavior, is a model of two parallel machines and a re-entrant product flow. The model is specified in the χ language. The behavior of this two-machine model has been analyzed by means of nonlinear time series analysis and sensitivity analysis.

Simulation of the two-machine model produces a time series of the interdeparture times of the products. Time series analysis of these interdeparture times produces no meaningful results; phase space reconstruction is impossible due to the fact that the elements of the time series take a limited number of recurring values.

Sensitivity analysis methods are used to find out if a system meets one of the defining characteristics of chaos: sensitivity to small differences in initial conditions. Initial conditions of discrete-event models are usually integers, which cannot be perturbed by a number smaller than one. A value that can be perturbed by a number smaller than one is the remaining work content of a machine. Since the remaining work content cannot be changed directly in a discrete-event model, a small amount of time is added to the remaining processing time of a single specific product. This new method of sensitivity analysis is applied to the model of the two-machine simulation model. Results indicate that the model reacts sensitive to specific small differences. In most cases, the model is not sensitive to small differences, so that the irregular behavior of the model is not chaotic in the strict sense as defined in the well established theory of chaos in dynamic systems.

Structural stability analysis is used to determine how parameter settings can affect structural changes in the behavior of the system. It is well known that changes in parameters such as scheduling rules and WIP-levels of a system can be used to improve the behavior of a system.

Chaotic behavior in models of *process type* production systems is well known in literature. The systems considered are liquid tanks with a switched flow server. Such models are hybrid. Chaotic behavior in models of *discrete* production systems was found in two kinds of models: hybrid models—where in many cases the flow of discrete products is approximated by a steady flow equal to the average flow-rate of the discrete products—and discrete-event models where the logistic map was used as part of the model. No chaotic behavior, as defined in the well established theory of chaos in dynamic systems, was found in other models of discrete production systems. The presence of chaotic behavior in a model of a discrete production system, however, only implies chaotic behavior in a *real* discrete production system if the model is a *valid* model of the production system. The question of the validity of the models of discrete production systems with chaotic behavior was not addressed in literature. In one case, where the WIP level of a discrete production system was modeled by means of the logistic map, it was easy to show that the model was in fact invalid. In literature, the irregular, apparently chaotic behavior of some small models of discrete production systems has been proved to be periodic. The period of this behavior, however, can be much longer than the observed time or the observed amount of products. In fact, the period can be given any length by adjusting model parameters. Realistic, non-artificial discrete-event models of discrete production systems that show chaotic behavior, as defined in the theory of chaos in dynamic systems, were not found in this study.

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